Appendix – Impedance calculation demonstration and its validity range for 2-port shunt-transmission setup of a VNA

In this appendix, the formula of a circuit's load impedance will be derived, when the load is connected with a VNA in shunt configuration.

Consider the setup in the figure:

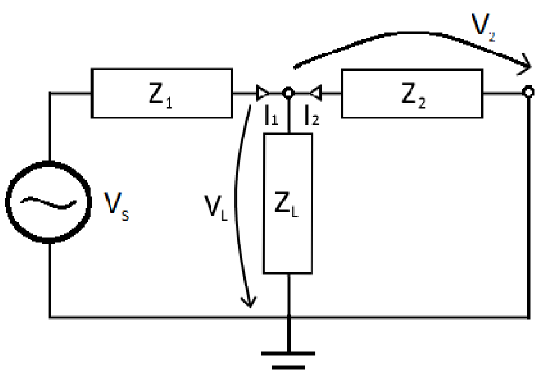

Figure 1 - Load impedance measurement by shunt-connection method

V_S represents the voltage generator of port 1 and V_2 the measured voltage at port 2. The potential differences indicated by the arrows are:

$$V_L = V_2 \qquad (1)$$

Since this is a simple voltage divider circuit, it is straightforward that V_L is

$$V_L = V_S \frac{\frac{Z_L Z_2}{Z_L+Z_2}}{Z_1+\frac{Z_L Z_2}{Z_L+Z_2}} \qquad (2)$$

and the currents are

$$I_1 = \frac{V_L}{\frac{Z_L Z_2}{Z_L+Z_2}} \qquad (3)$$

$$I_2 = -\frac{V_L}{Z_2} \qquad (4)$$

The above-mentioned configuration is achieved by connecting the VNA to the DUT as shown below:

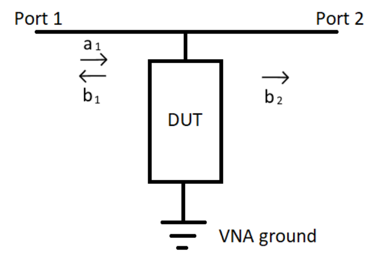

Figure 2 - Shunt-connection of a DUT to a VNA

a and *b* are the travelling waves described from [1]:

$$a_1 = \frac{\sqrt{|\mathrm{Re}(Z_1)|}}{2|Z_1|}(V_L + Z_1 I_1) \qquad (5)$$

$$b_2 = \frac{\sqrt{|\mathrm{Re}(Z_2)|}}{2|Z_2|}(V_2 - Z_2 I_2) \qquad (6)$$

Their ratio is the S-parameter S_{21}:

$$S_{21} = \frac{b_2}{a_1} = \frac{\frac{\sqrt{|\mathrm{Re}(Z_2)|}}{2|Z_2|}(V_2 - Z_2 I_2)}{\frac{\sqrt{|\mathrm{Re}(Z_1)|}}{2|Z_1|}(V_L - Z_1 I_1)} \qquad (7)$$

which, by substituting the (3) and (4) in (7) becomes:

$$S_{21} = \sqrt{\left|\frac{Re(Z_2)}{Re(Z_1)}\right|}\left|\frac{Z_1}{Z_2}\right| \frac{V_1 + Z_2 \frac{V_1}{Z_2}}{V_1 + Z_1 \frac{V_1}{\frac{Z_L Z_2}{Z_L+Z_2}}} \qquad (8)$$

and using the (1) and (2) in (8):

$$S_{21} = \sqrt{\left|\frac{Re(Z_2)}{Re(Z_1)}\right|}\left|\frac{Z_1}{Z_2}\right| \frac{2\, Z_L Z_2}{Z_L(Z_1+Z_2)+Z_1 Z_2} \qquad (9)$$

Thus, if (9) is solved by the load impedance:

$$Z_L = \frac{Z_1 Z_2 S_{21}}{2\left(Z_2\left|\frac{Z_1}{Z_2}\right|\sqrt{\left|\frac{Re(Z_2)}{Re(Z_1)}\right|} - S_{21}\frac{Z_1+Z_2}{2}\right)} \qquad (10)$$

Finally, under the assumption that the port impedances are identical (it is at this point necessary to assume some error in this calculation due to non-ideality of the ports' and probes' matching):

$$Z_0 = Z_1 = Z_2 \qquad (11)$$

it is possible to simplify the (10):

$$Z_L = \frac{Z_0}{2}\frac{S_{21}}{(1-S_{21})} \quad (12)$$

The calculated of Z_L is subject to uncertainty due to the VNA being an instrument with finite precision.

In order to calculate the uncertainty for formula (12), the following transmission accuracy charts in the datasheet of R&S®ZNL shall be taken in consideration:

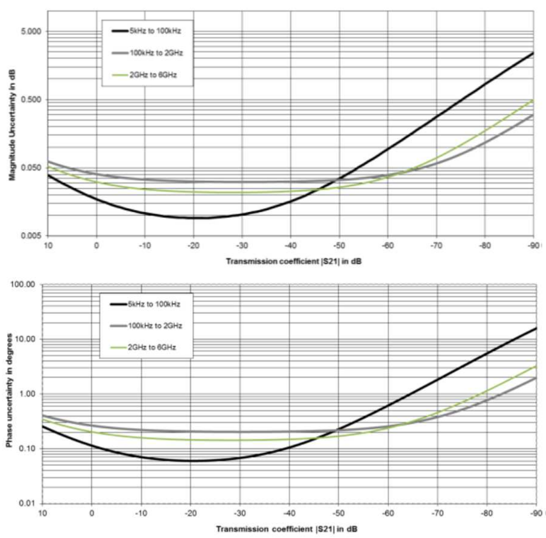

Figure 3 – Transmission measurement accuracy of R&S®ZNL. Magnitude (above) and phase (below).

Notice that these curves are valid for devices calibrated with the settings specified in the datasheet, therefore conditional to use of R&S® ZV-Z270 and TOSM calibration and measurement with -10dBm power and 10 Hz IFBW.

In order to understand how the R&S® ZNL performs, let the accuracy worst-cases amongst the values for the whole frequency range be considered.

One is obtained by S_{21} = -90 dB ≃ 0.0000316. At that point the magnitude uncertainty is 2.5 dB and phase uncertainty of 15°. By leaving the phase out of our calculation (therefore introducing error again), and assuming Z_0 = 50 Ω, equation (12) yields:

$$Z_L = 25\frac{0.0000316}{(1-0.0000316)} \simeq 790\mu\Omega \quad (13)$$

A variation of ±2.5 dB results in:

$$Z_L = 25\frac{0.0000237}{(1-0.0000237)} \simeq 592\mu\Omega \quad (14)$$

$$Z_L = 25\frac{0.0000422}{(1-0.0000422)} \simeq 1055\mu\Omega \quad (15)$$

Which means that there is roughly -25% to +33.5% uncertainty (a conservative ±50% may be assumed because of the errors neglected up to this point) in the measurement of a 790 µΩ impedance between 5 kHz and 100 kHz.

On the other hand, another worst-case occurs (for passive DUTs the received wave amplitude cannot be higher than the one sent) for S_{21} → 0 dB = 1 in the frequency range from 2 GHz to 6 GHz. A S_{21} = -1 dB ≃ 0.89 yields an uncertainty of 0.04 dB magnitude and 0.25° phase (much smaller than the previous case, thus not representing a performance limit of the VNA). Similarly to (13), (14) and (15):

$$Z_L = 25\frac{0.8912509}{(1-0.8912509)} \simeq 204.9\Omega \quad (16)$$

A variation of ±0.04 dB results in:

$$Z_L = 25\frac{0.887156}{(1-0.887156)} \simeq 196.5\Omega \quad (17)$$

$$Z_L = 25\frac{0.8953647}{(1-0.8953647)} \simeq 213.9\Omega \quad (18)$$

For a shunt-transmission setup, this demonstrates that measuring lower impedances at lower frequencies is the only limit of the R&S® ZNL.

For the reflection setup, the formula to calculate the impedance is:

$$Z_L = Z_1 \cdot \frac{1+S_{11}}{1-S_{11}} \quad (19)$$

The uncertainty for this setup follows a procedure similar to the one explained before. For this configuration however, the measurement accuracy charts to be considered shall be those for reflection measurements.

References

[1] R. B. Marks und D. F. Williams, "A General Waveguide Circuit Theory", J. Research of the National Institute of Standards and Technologies, Vol. 97, No. 5, September-October 1992

[2] Rohde & Schwarz “R&S®ZNL Vector Network Analyzer – Datasheet V03.00”, November 2017, *https://www.rohde-schwarz.com/de/broschuere-datenblatt/znl/* (requested March 15, 2019)