Theory and implementation of a precision lock-in frequency meter

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1、 Background
2、 Theory and simulation
3、 Implementation results
4、 Summary

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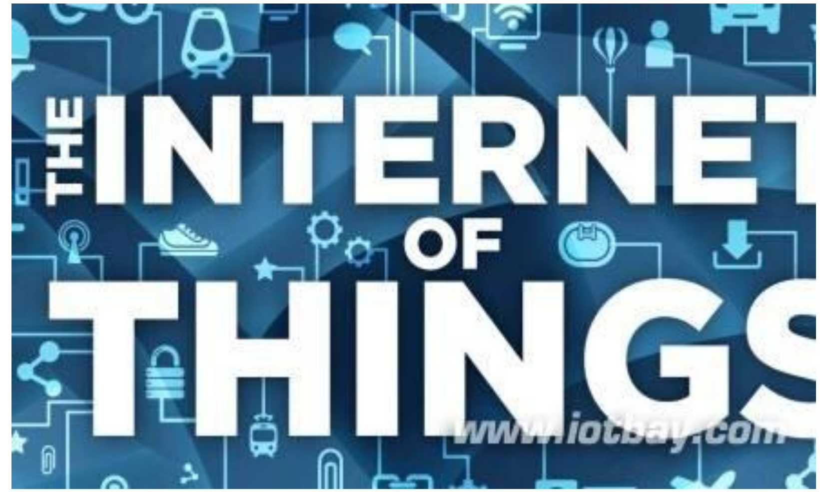
1.1 why precision frequency measurement?

Better GPS Positioning

Higher 5G capacitance



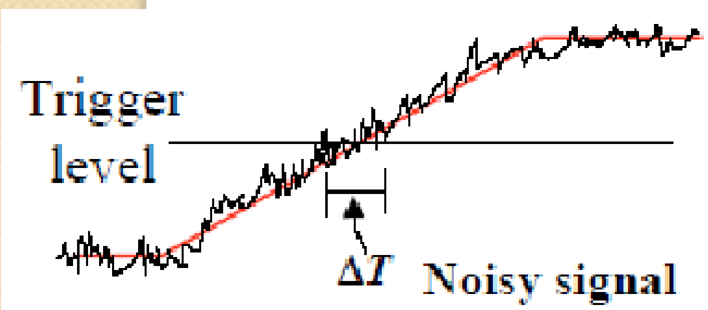
Ultra precise scientific experiments

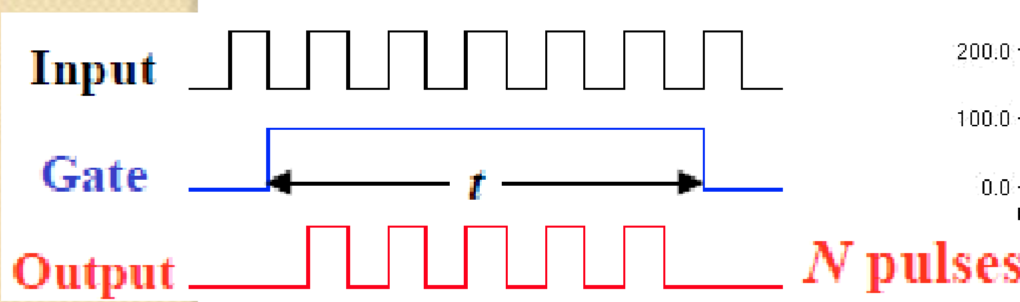
Finer control and fabrication

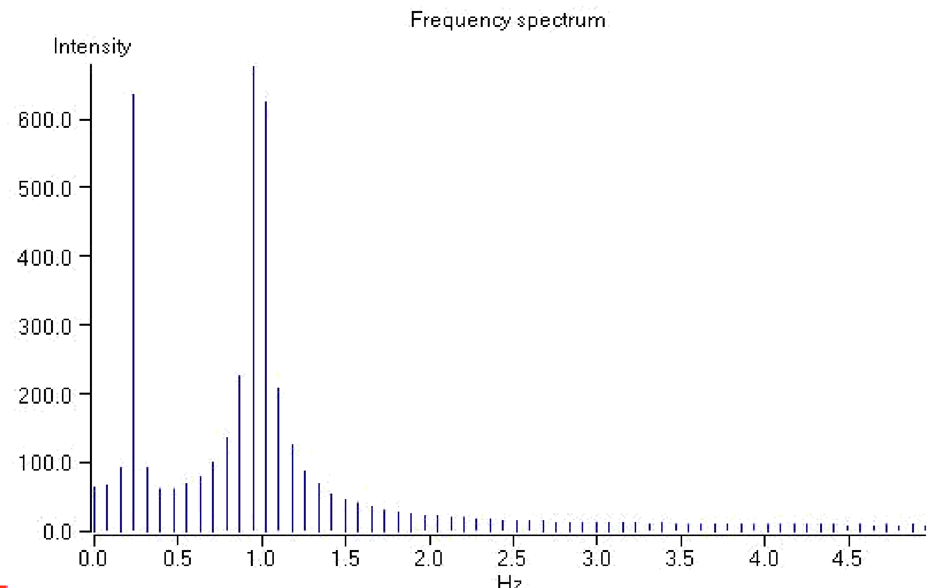
1.2 Long history of frequency measurement

$$f = \frac{N}{t}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$







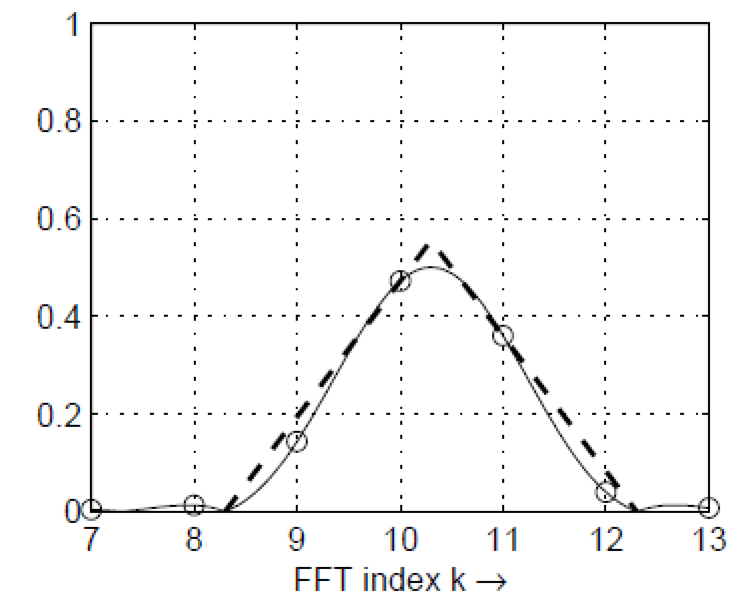
In time domain: to counter and measure duration.
(Only for high SNR)

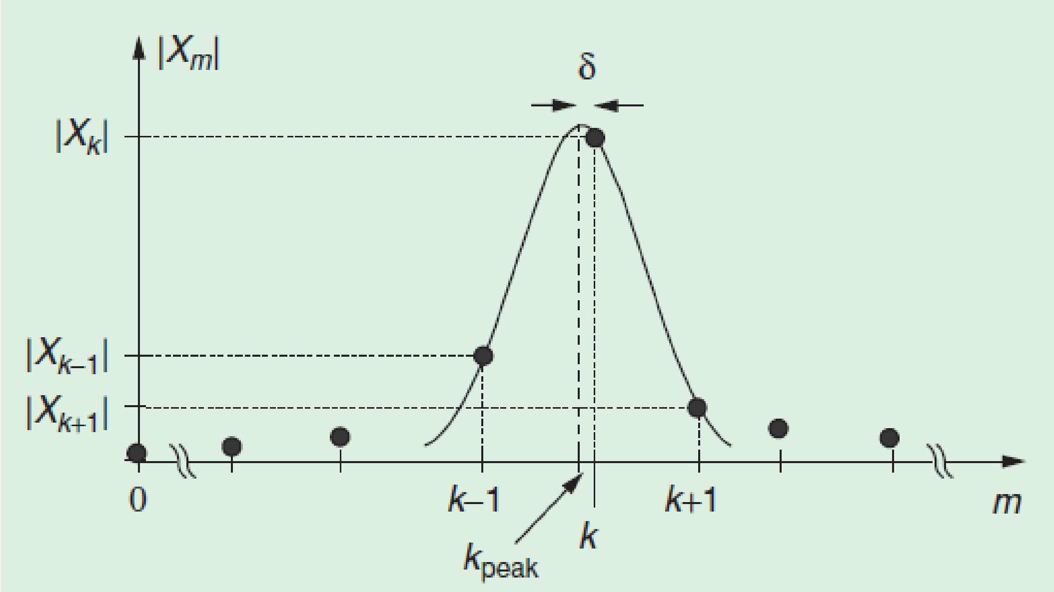
In frequency domain: Fourier transformation.
(Only for coarse estimation)

1.3 To push the precision: previous work(1/3)

Triangle function fitting
the 1st order approximation

Parabolic function fitting:
the 2nd order approximation



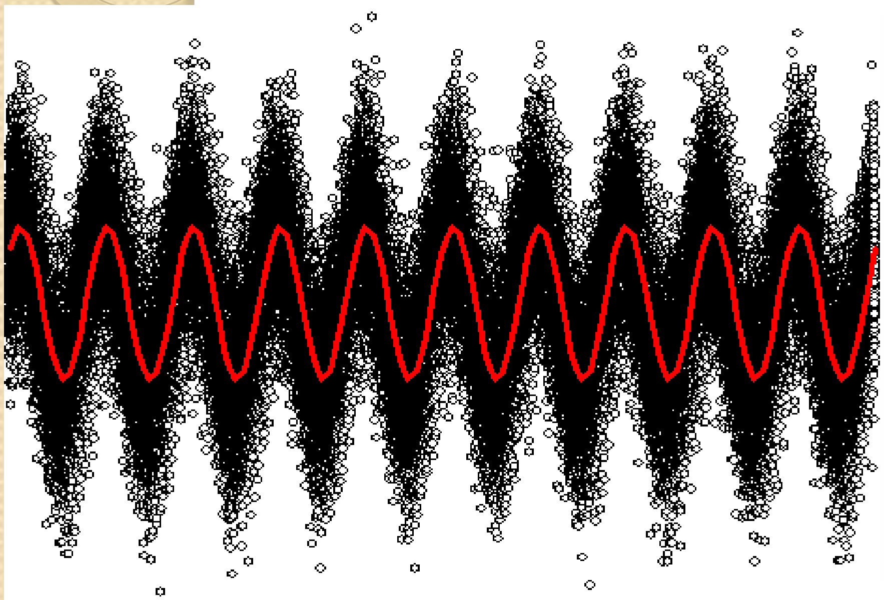


Eric Jacobsen and Peter Kootsookos, Fast accurate frequency estimators, IEEE Sig. Proc. Mag., 2007, 123(2007)

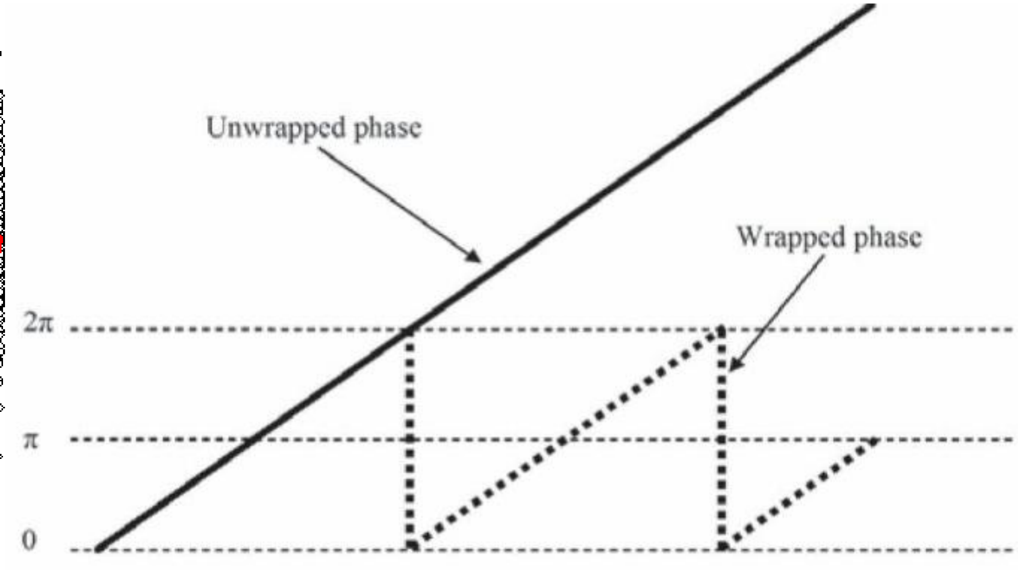
K. Hofbauer, Estimating frequency and amplitude of sinusoids a survey and the use of shifted Fourier transforms, Diploma Thesis, Graz Univ. Tech., 2004

1.4 To push the precision: previous work(2/3)

Maximum likelihood estimation



Phase unwrapping



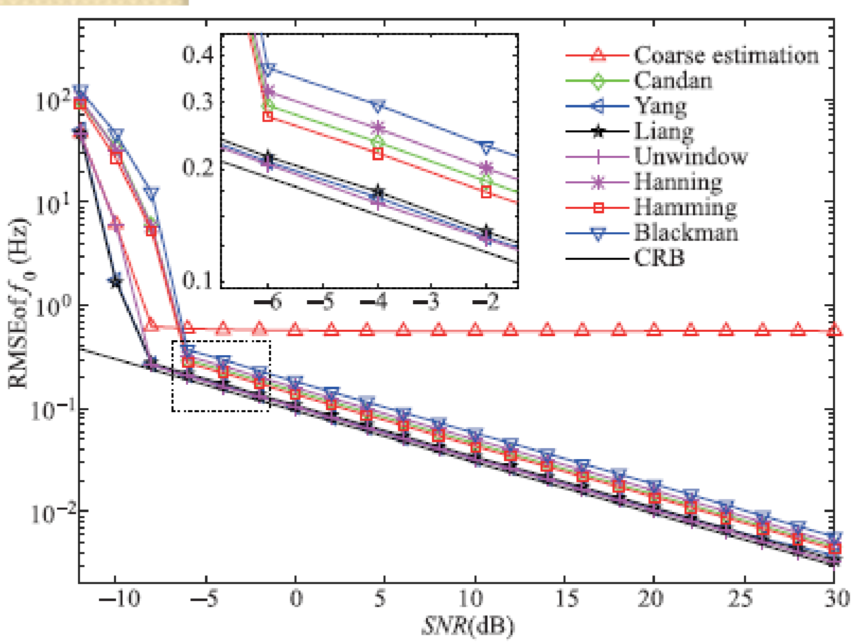
Z. Shen and S. Zhang, Improved frequency estimation algorithm by least squares phase unwrapping, Circuits Syst. Signal Proc., 37, 5680-7(2018)

D.C. Rife and R.R. Boorstyn, Multiple tone parameter estimation from discrete-time observations, Bell Sys. Tech. J., 55(9), 1389-410(1976)

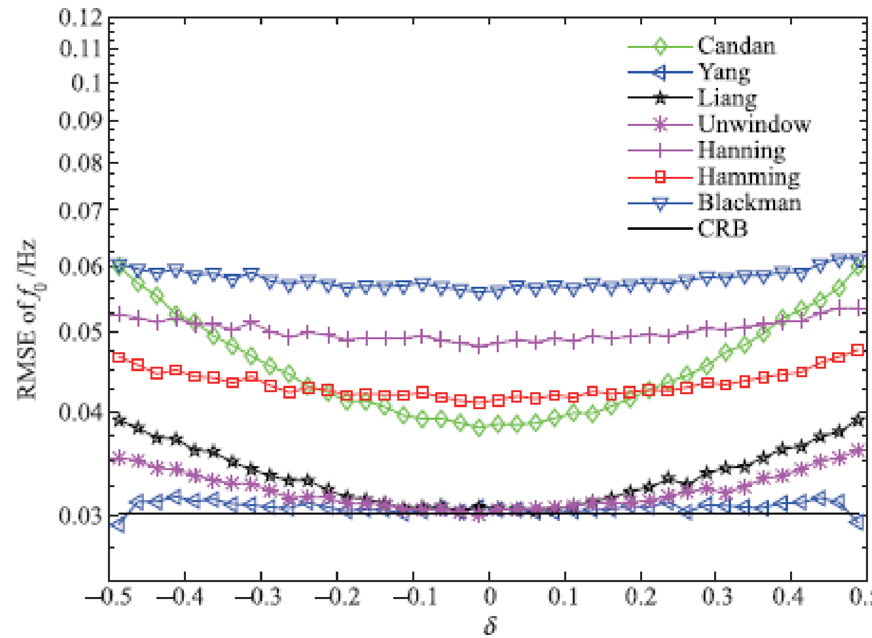
1.5 To push the precision: previous work(3/3)

Cramer-Raw Lower Bound

$$var(\omega) = \frac{12\sigma^2}{{b_0}^2 T^2 N(N^2 - 1)}$$



Windows optimization



Unfortunately, getting frequency at low SNR with proper results, still remains a challenge.

J.Z. Xiang, W. Cui and Q. Shen, Chin. J. Electr., 27(1), 109-14(2018)

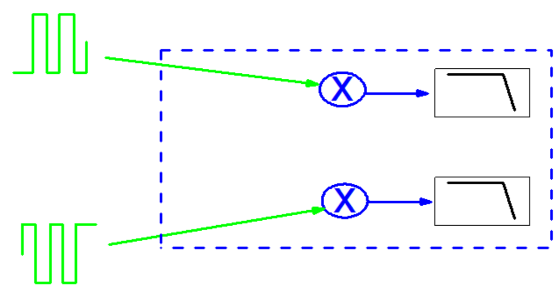
D.C. Rife and R.R. Boorstyn, IEEE Trans. Inform. Theor., 20, 591-8(1974)

2.1 Concepts of the lock-in amplifiers(LIAs)

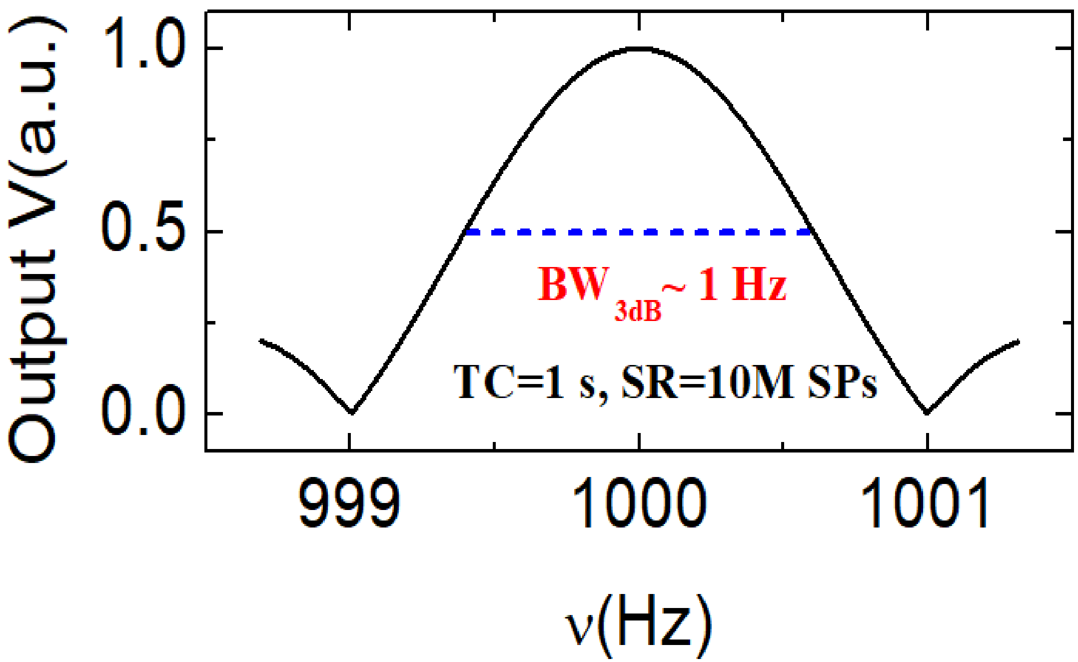
In principle, LIAs work as homodyning mixers for phase sensitive detection(PSD)

As an ultra narrow band pass filter

$$S_{\mathrm{Re}}(\varpi) = \frac{2}{T}\int_{t=0}^{T}[S(t)\times\cos(\varpi t)]\mathrm{d}t$$



$$S_{\mathrm{Im}}(\varpi) = \frac{2}{T}\int_{t=0}^{T}[S(t)\times\sin(\varpi t)]\mathrm{d}t$$

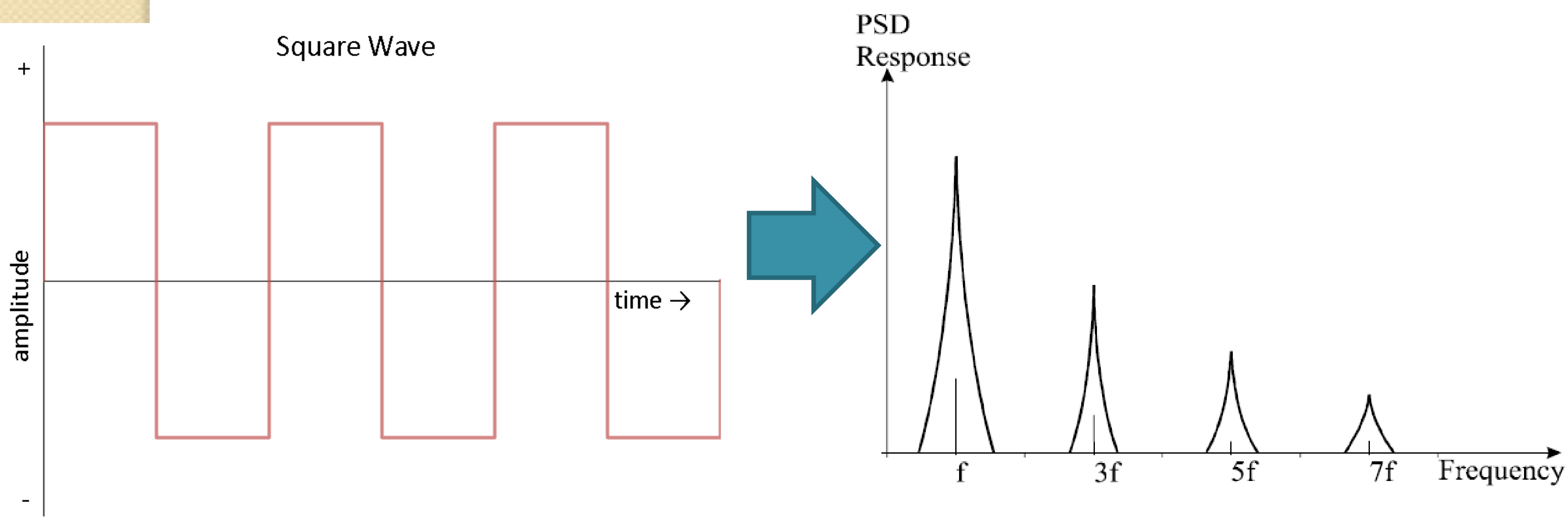


LIAs can easily separate in-phase and quadrature phase components

M. L. Meade, Lock-In Amplifiers: Principles and Applications. (Peter Peregrinus, London, 1983)

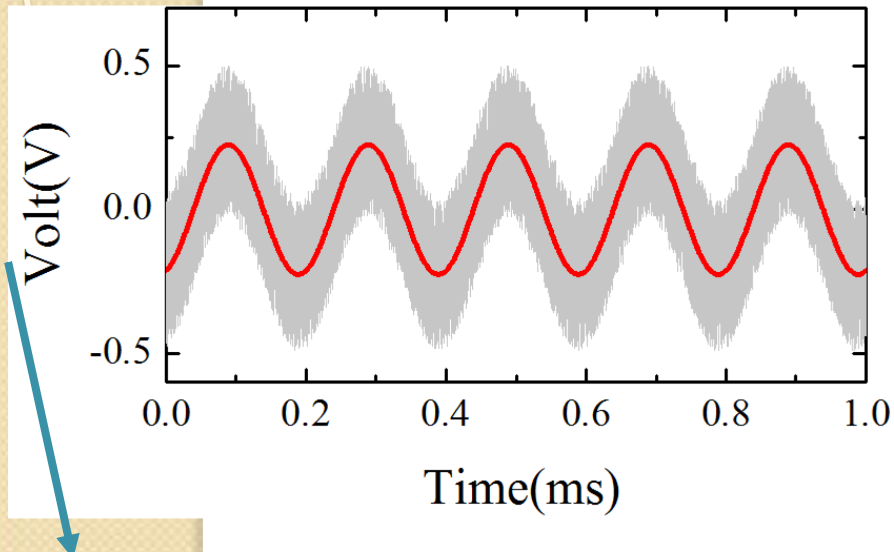
2.2 Classic LIAs work at fixed frequencies

Single reference input is normally required.



With switches, other frequencies such as harmonic ones could also be measured(heterodyning mode).

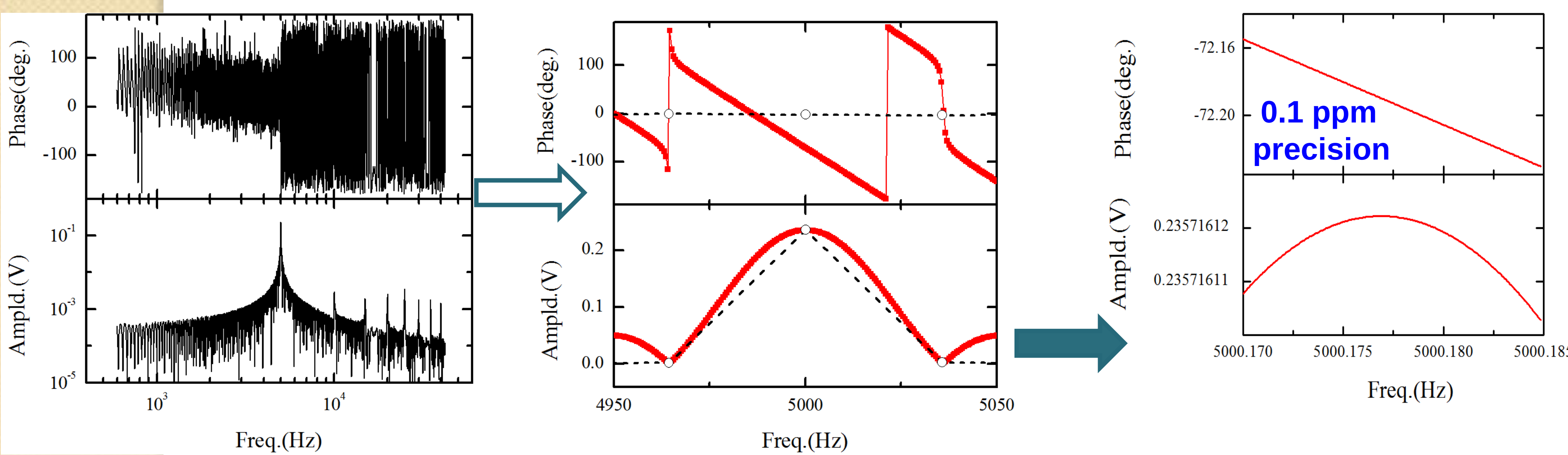
2.3 Lu's Fine scanning LIAs: lock-in frequencymeter LIFs



Based on **correlation functional theory**

$$R_{ij} = \frac{1}{N}\sum_{k=0}^{N}[S_k \times S_{ref,k}(f_i, \Psi_j, 1)]$$

FFT could not finely tuned in frequency range.



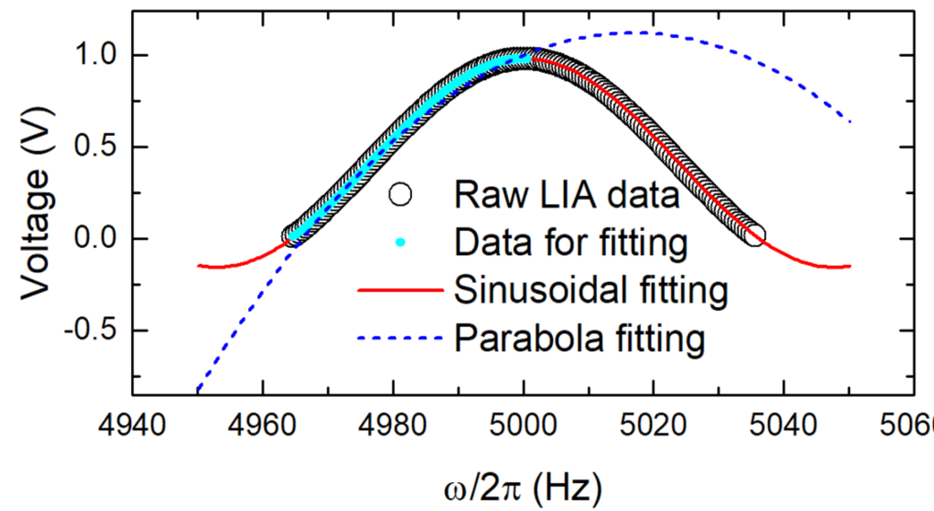
Frequency taken as a variable, and no reference input needed.

2.4 Theory on the local spectrum of LIFs

$$LIF(\Delta\omega)_{RE} = \frac{2}{T_0}\int_0^{T_0=\frac{2\pi}{\omega_0}} \cos(\omega_0 t)\sin[(\omega_0+\Delta\omega)\mathrm{t}]\,dt = \frac{2\omega_0(\omega_0+\Delta\omega)\sin\left(\frac{\pi\Delta\omega}{\omega_0}\right)^2}{\Delta\omega\pi(2\omega_0+\Delta\omega)}$$

$$LIF(\Delta\omega)_{IM} = \frac{2}{T_0}\int_0^{T_0=\frac{2\pi}{\omega_0}} \sin(\omega_0 t)\sin[(\omega_0+\Delta\omega)\mathrm{t}]\,dt = \frac{2\omega_0{}^2\sin\left(\frac{\pi\Delta\omega}{\omega_0}\right)cos\left(\frac{\pi\Delta\omega}{\omega_0}\right)}{\Delta\omega\pi(2\omega_0+\Delta\omega)}$$

$$|LIF(\Delta\omega)| = \sqrt{LIF(\Delta\omega)_{RE}{}^2 + LIF(\Delta\omega)_{IM}{}^2}$$



$\boldsymbol{when\ \Delta\omega \ll \omega_0,}$

$$|\boldsymbol{LIF}(\boldsymbol{\Delta\omega})| \cong \left|\frac{2\omega_0{}^2\sin\left(\frac{\pi\Delta\omega}{\omega_0}\right)cos\left(\frac{\pi\Delta\omega}{\omega_0}\right)}{\Delta\omega\pi(2\omega_0+\Delta\omega)}\right| \approx \frac{2\omega_0 cos\left(\frac{\pi\Delta\omega}{\omega_0}\right)}{(2\omega_0+\Delta\omega)} \simeq \boxed{cos\left(\frac{\pi\Delta\omega}{\omega_0}\right)}$$

Strictly, trigonometric function is better than triangle and parabolic ones.

2.5 work process of LIFs

Fine scanning LIAs → **Local frequency spectrum(>2 points)**

↓

Initiate parameters → **Especially** $\omega'_{Init} = \frac{N}{2SR}$

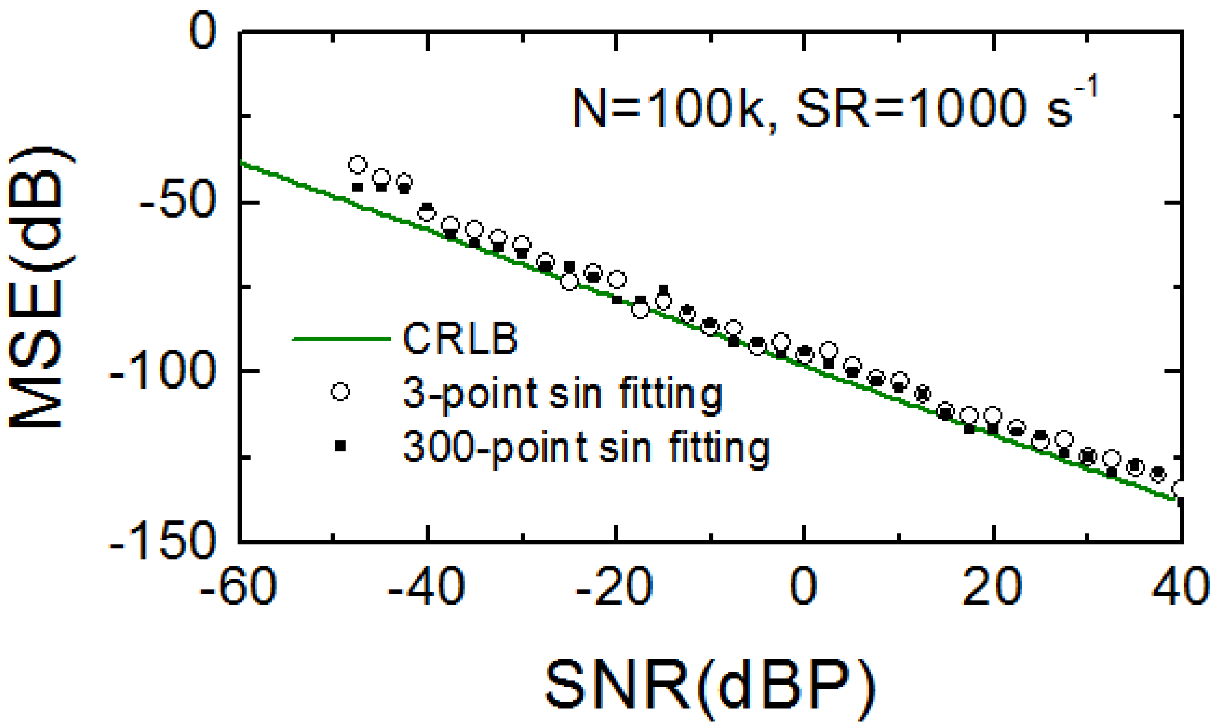
↓

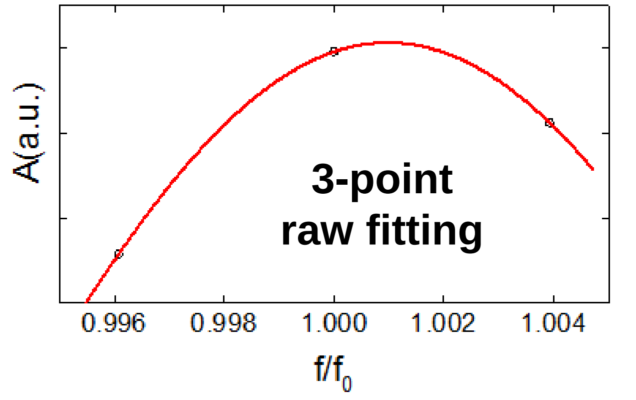
Peak detection with sinusoidal fitting → **Local maximum f_m**

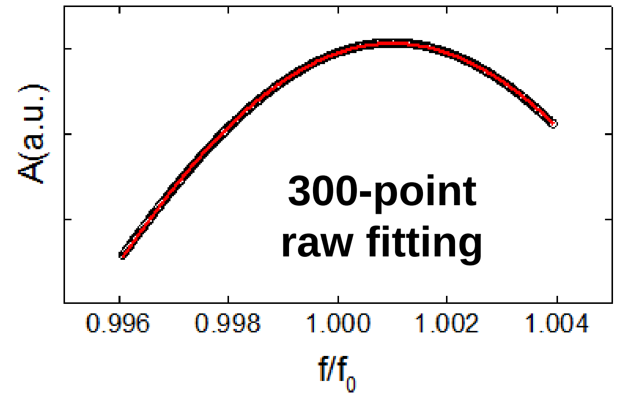
↓

Normal LIA with $\mathbf{f_m}$ → **Get V_{rms} and phase**

2.6 3-point sinusoidal fitting is enough

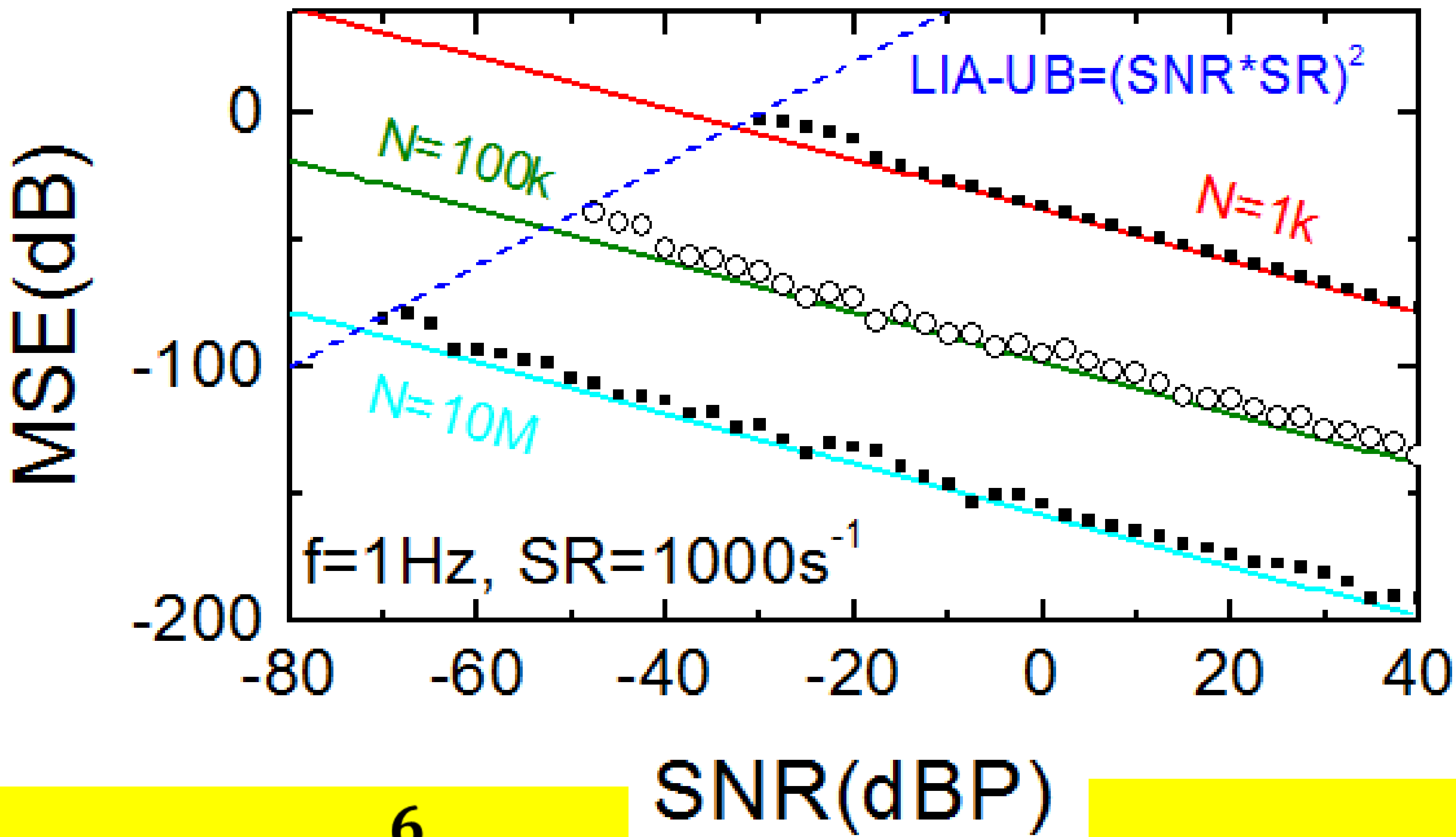






Calculation complexity in total ~ 6N for one turn(multiplication and plus)

2.7 Performance simulation of LIFs(1)

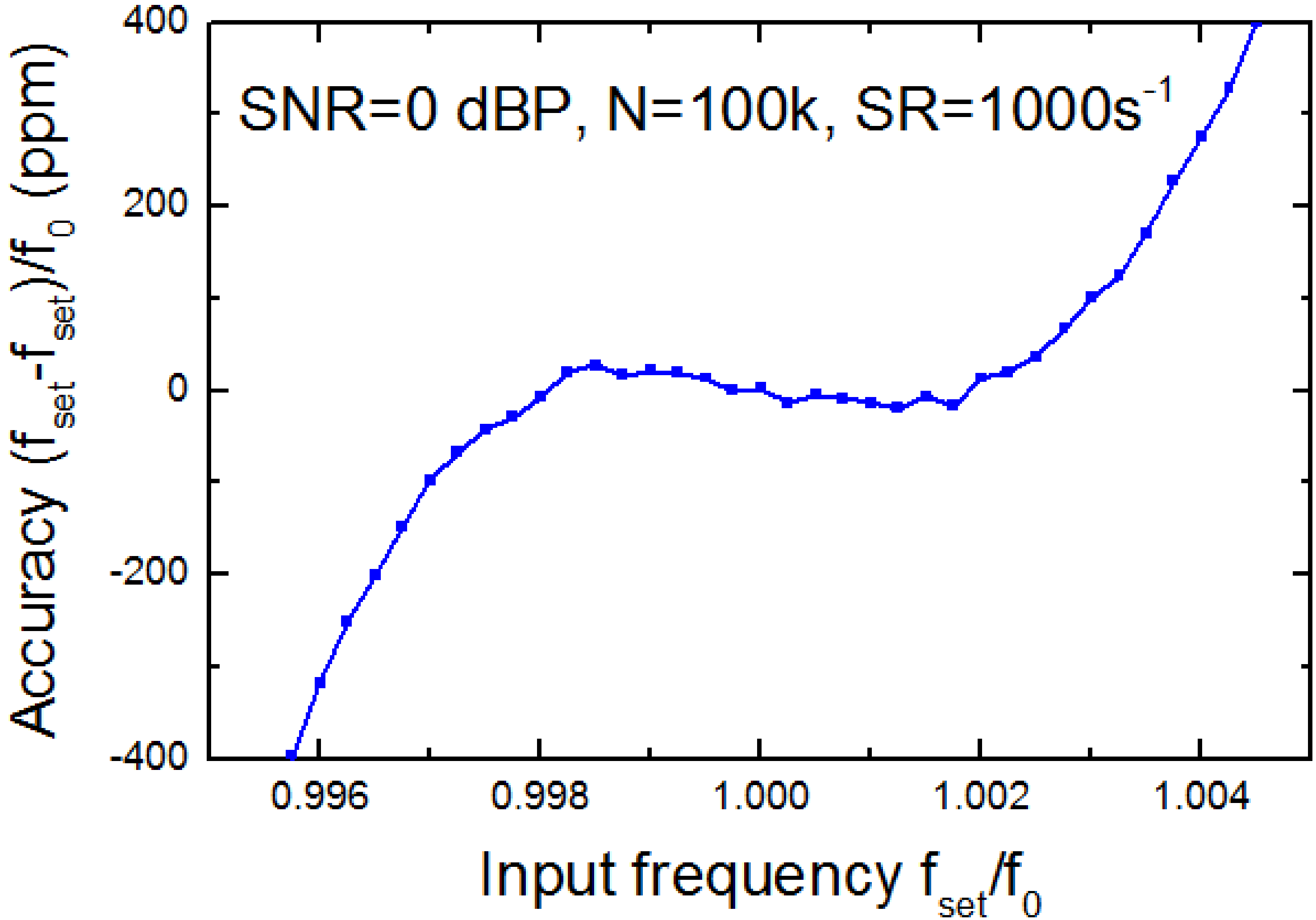


$$var(f)_{LB} \approx \frac{6}{(2\pi)^2 R_p T^2 N^3}$$

$$var(f)_{UB} \approx \frac{R_p}{T^2}$$

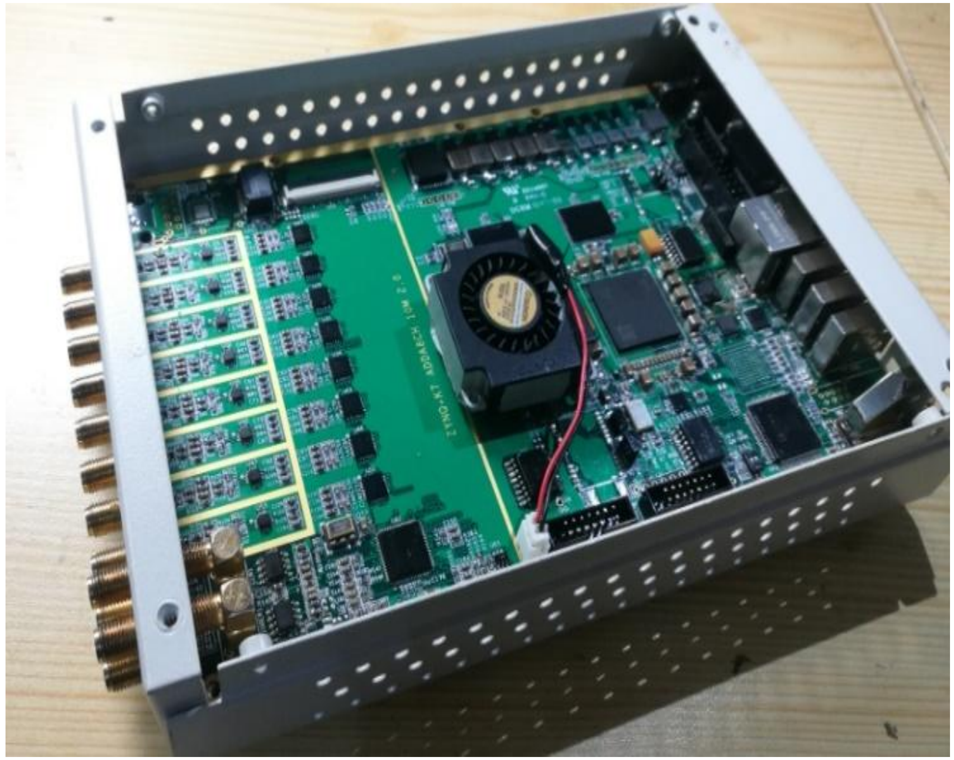
Well agree to the Cramer-Rao lower bound(CRLB) and LIA upper bound

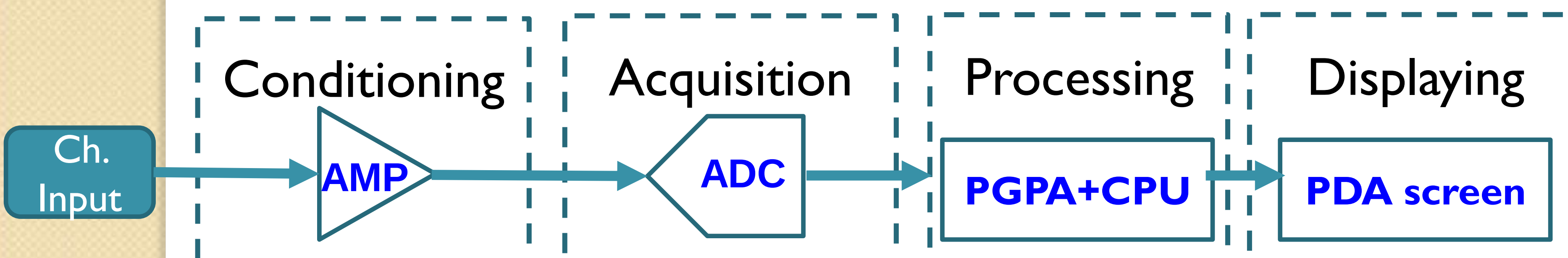
2.8 Performance simulation of LIFs(2)



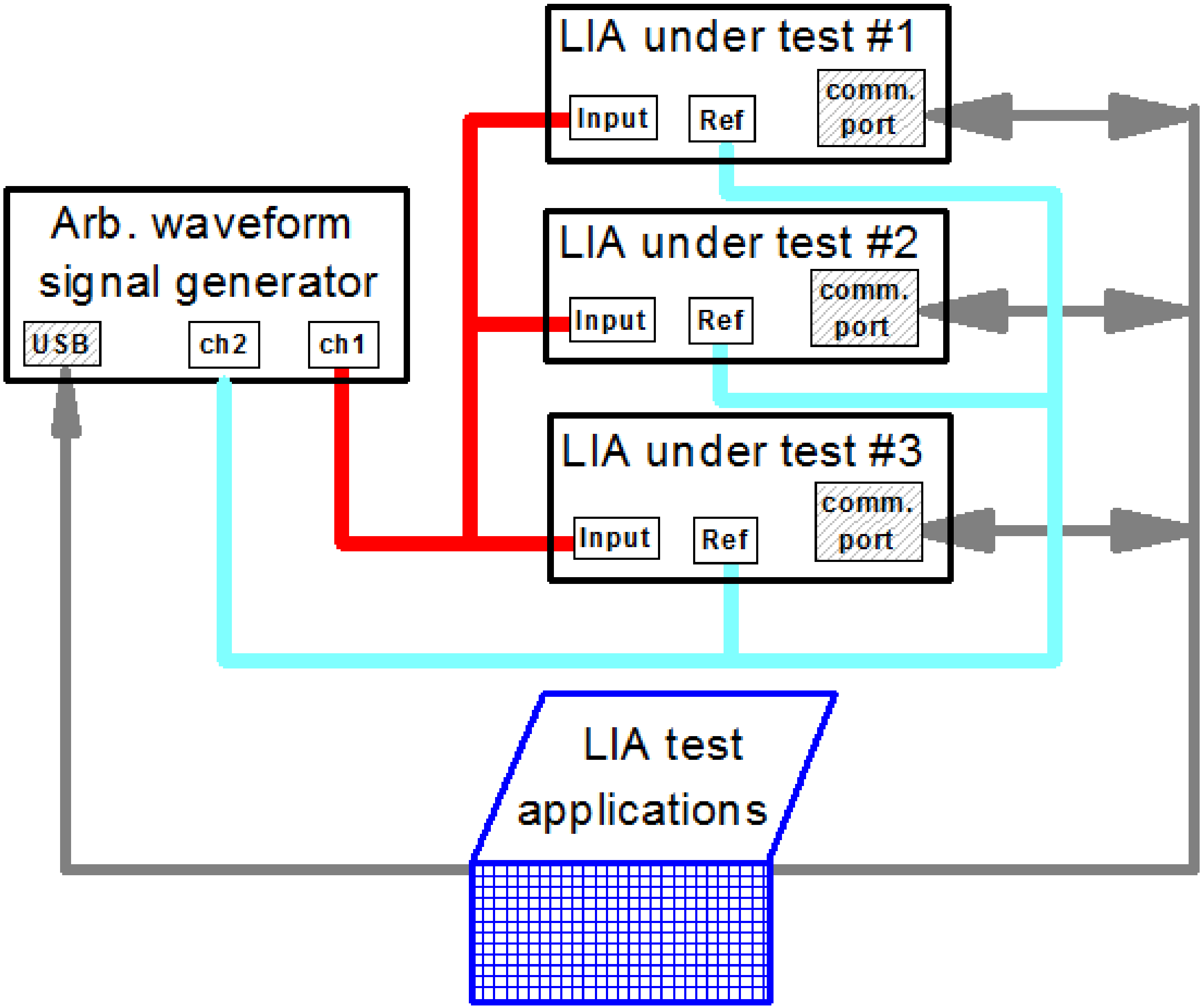
Frequency accuracy at lower SNR is reasonable high.

3.1 Implementation and appearance

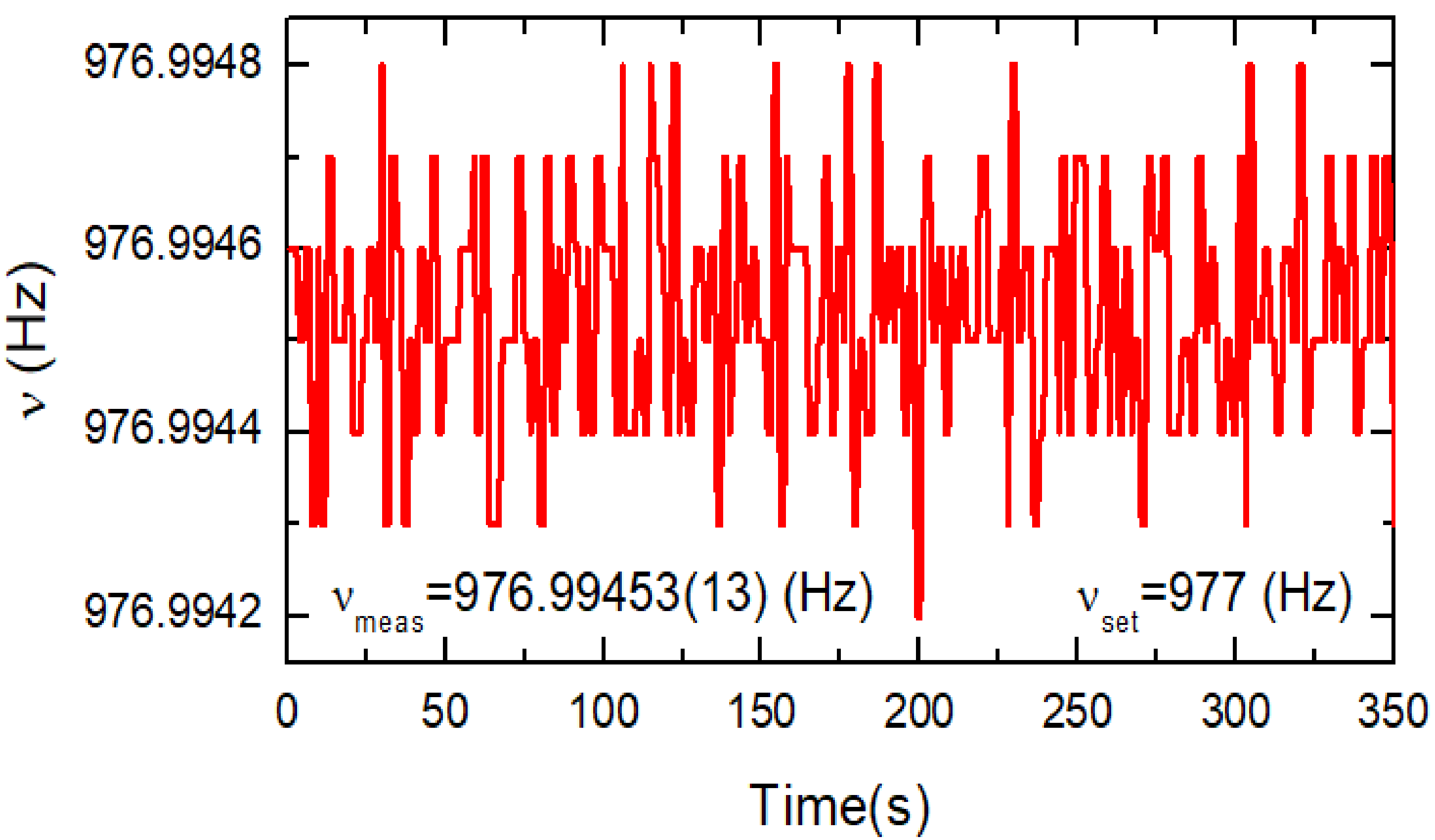




3.2 Test setup

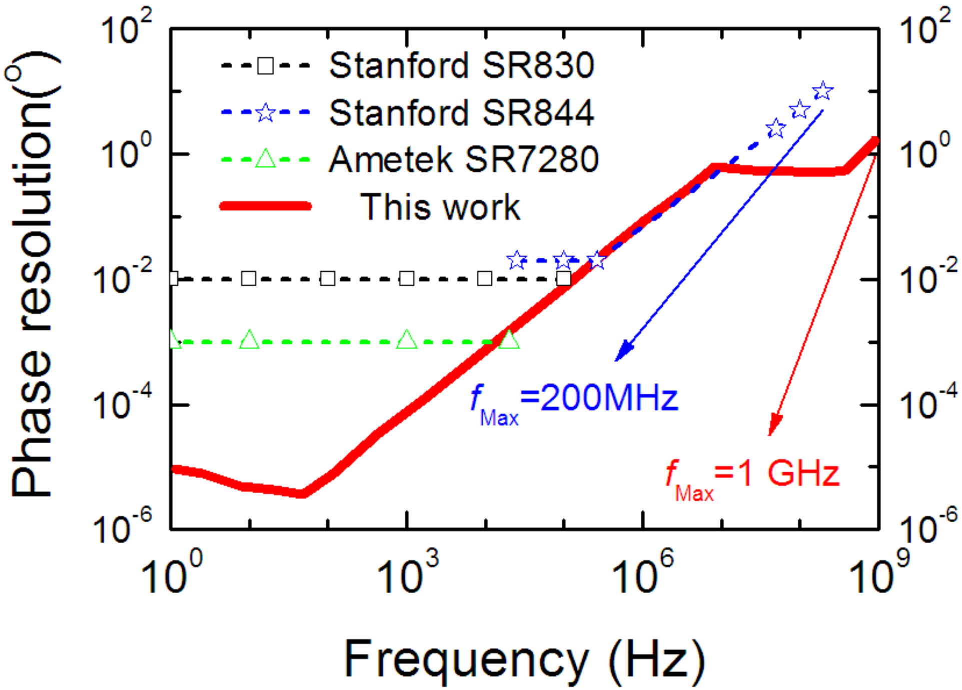


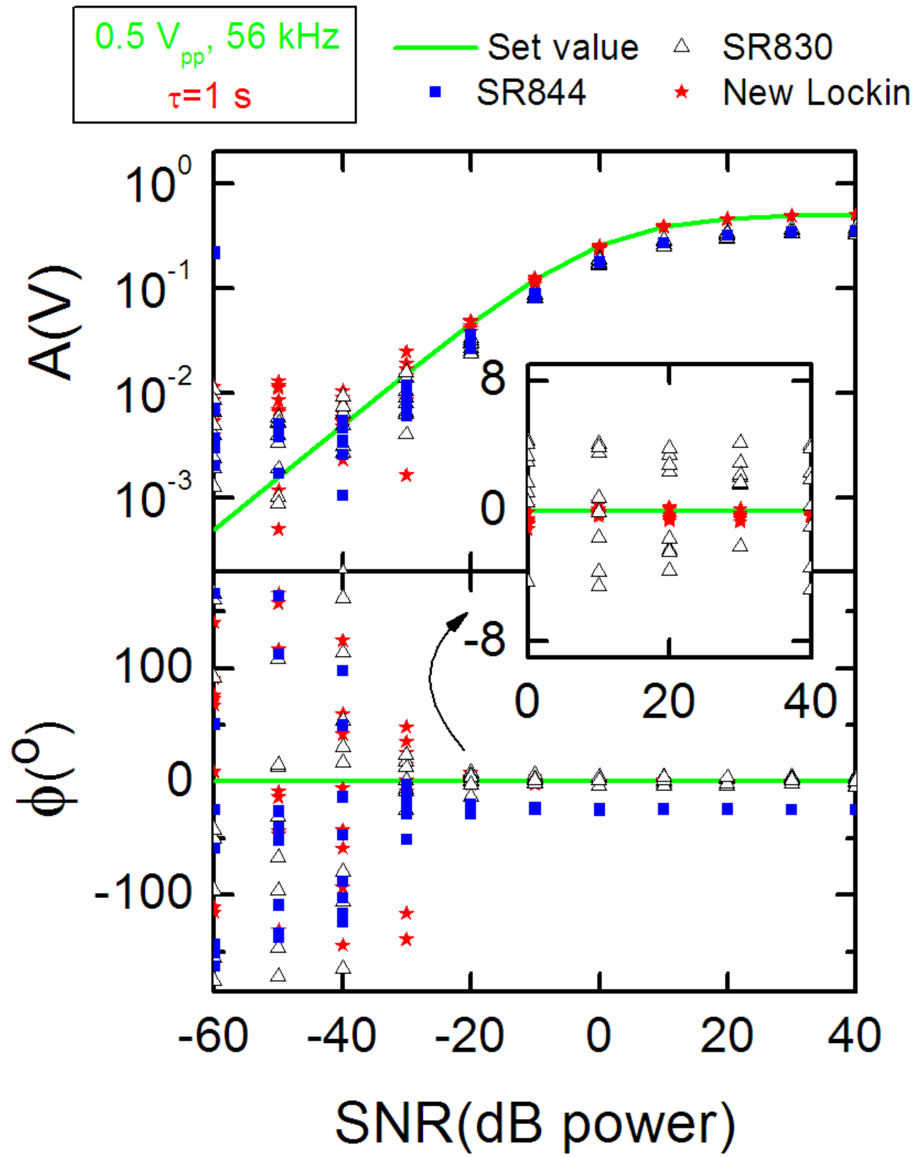
3.3 Measurements for real signal.



Real measurement of a 977 Hz signal with 130 micro Hz uncertainty.

3.4 Comparison of amplitude/phase/bw





Bandwidth、precision、and phase resolution is better than some commercial LIAs

4. Summary of Lu's LIFs

1、**Principle innovation** **in both frequency measurement and LIAs.**

2、**Better fitting function** **of local frequency.**

3、Std. dev. Approach **CRLB with O(N)** cost.

4、Precision as good as **0. 2 ppm.**

$f = \frac{N}{t}$ $\quad F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$ **Important formula at one glance**

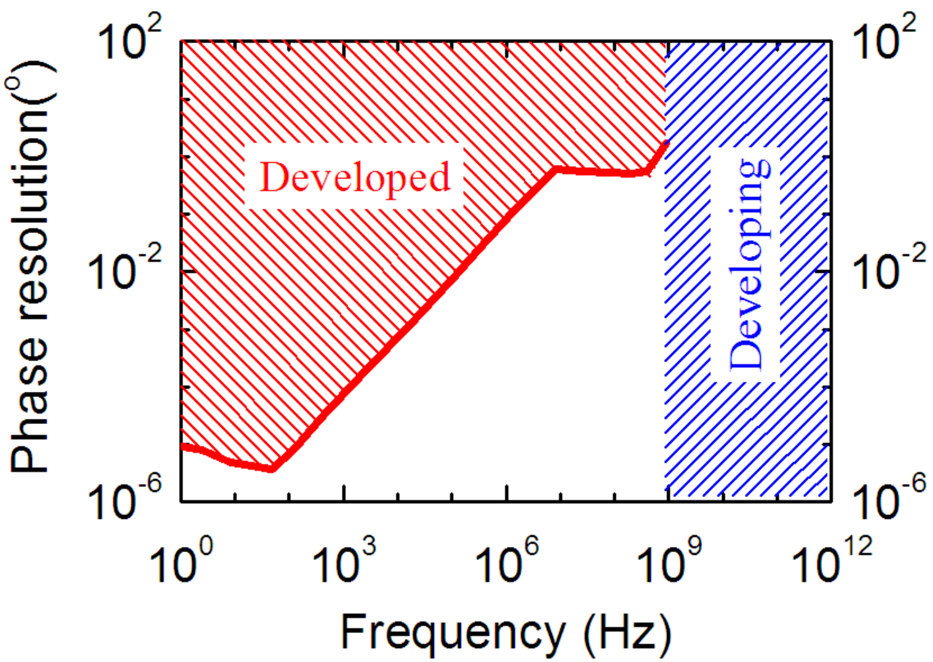
$R_{ij} = \frac{1}{N}\sum_{k=0}^{N}[S_k \times S_{ref,k}(f_i, \Psi_j, 1)]$ $\quad cos\left(\frac{\pi\Delta\omega}{\omega_0}\right)$ $\quad \omega'_{Init} = \frac{N}{2SR}$ $\quad var(f)_{UB} \approx \frac{R_p}{T^2}$ $\quad var(f)_{LB} \approx \frac{6}{(2\pi)^2 R_p T^2 N^3}$

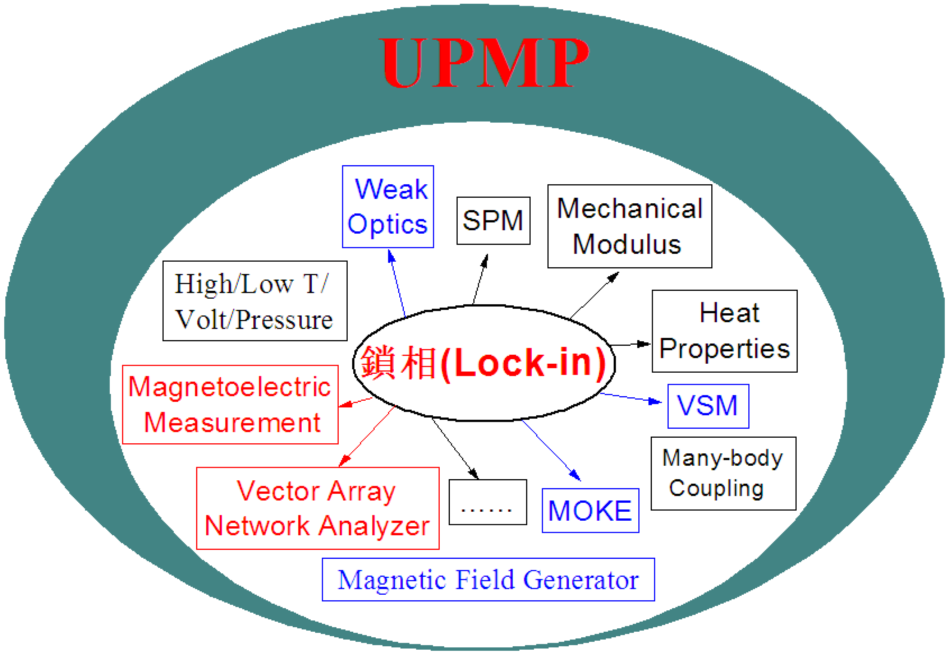
Acknowledgement

N S F C

Research 研创达
高速数据采集专家

Thanks for your attention!





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We R&D and produce precision broadband instruments MADE IN CHINA.